

Earthquake Risk Assessment by Using Collective Risk Models

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ABSTRACT

In this study, we aimed to determine and evaluate the risk uncertainties in natural disaster planning especially for earthquakes by using collective risk model (CRM). We investigated destructive earthquakes in the world having magnitude 7.5 or greater and damage amount approximately \$1 million or more between 1980 and 2014. The total amount of earthquake damage is calculated after the distribution of the number of earthquakes is determined to be Poisson, and the amount of damage is log-normal. The results are evaluated for various risk loading factors by using the premium principle method based on the expected value principle (EVP) and the standard deviation (SDP) principle. As a result of this study, for small risk loading factors such as 0.1 and 0.2, the minimum total amount of earthquake damages in the world are determined by using EVP and SDP principles. As the risk loading factors increase, total amount of earthquake damages in the world in magnitude 7.5 or greater are beginning to increase significantly.

KeyWords: Earthquake, collective risk model, expected value principle, standard deviation principle.

INTRODUCTION

Today, countries are actively evaluating the risks in order to achieve economic stability in their development by taking a number of measures. But the events that cannot be predicted before it occurs, such as natural disasters, cause insufficient risk assessment. Disaster planning is an approach that offers a solution for unpreventable natural disasters that will affect the societies negatively.

In the literature there are many studies about earthquake risk assessment. Erdik et al. (2003) evaluated earthquake risk scenario in Istanbul based on intensities and spectral displacements. Tsai and Chen (2010) focused on the risk assessment and management of earthquake disaster in the aspect of tourism in Taiwan where earthquakes frequently occur. Marulanda et

al. (2013) used comprehensive approach to probabilistic risk assessment for risk estimation in Barselona, Spain. Also some of the studies about collective risk models in the literature can be given as Meyers (2009), Hernández-Bastida and Fernández-Sánchez (2012), Cai and Tan (2007).

In this study, we aimed to determine and evaluate the earthquake risk uncertainties in natural disaster planning especially for earthquakes occurred between 1980 and 2014 by using collective risk model (CRM). The expected value and the standard deviation principle techniques are used for premium calculations with various loading factors when the earthquake damages magnitudes are 7.5 or greater and damage amounts are approximately \$1 million or more.

MATERIALS AND METHODS

Risk theory gives useful results in the ultimate decision-making process and provides evaluation by using mathematical analysis of the random fluctuations risk. Calculating the risk creates an opportunity and provides the more accurate evaluation for a future choice of a decision maker (Charpentier, 2015).

Let S be a random variable representing the total earthquake damage amount in the world in millions of dollars in a fixed period of time between 1980 and 2015. Let N be a random variable denoting the random number of earthquakes in the world. N is assumed to be Poisson distributed because Poisson distribution is the most suitable distribution for modelling occurrence of rare events such as natural disasters.

Let X_1, X_2, \dots, X_N be independent and identically distributed random variables representing each earthquake damage amount in this period and assumed to be log-normal distributed. N is independent from earthquake amounts X_1, X_2, \dots, X_N . S is modelled as the following form having compound Poisson distribution;

$$S = X_1 + X_2 + \dots + X_N \quad (1)$$

Collective risk model for S provides risk assessment for modelling damage amount and frequency of earthquakes.

The mean (expected value) and the variance for X by using log-normal distribution are as follows, respectively (Ross, 2014);

$$E(X) = e^{\mu + \sigma^2/2} \quad (2)$$

$$Var(X) = (e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$$

The mean for S by using the conditional distribution of S , given N are as follows (Boland, 2007);

$$\begin{aligned} E(S) &= E_N(E(S/N)) \\ &= \sum_{n=0}^{\infty} E(X_1 + \dots + X_N / N = n) P(N = n) \\ &= \sum_{n=0}^{\infty} [nE(X)] P(N = n) \\ &= E(X) \sum_{n=0}^{\infty} n P(N = n) \\ &= E(X)E(N) \end{aligned} \quad (3)$$

From Eq.(3), it can be easily seen that the expected value of the total earthquake damage is equal to the expected value of the earthquake damage amount times expected value of the number of earthquakes.

The variance for S by using the conditional distribution of S , given N are as follows (Boland, 2007);

$$\begin{aligned} Var(S) &= Var_N(E(S/N)) + E_N(Var(S/N)) \\ &= Var_N(E(X)N) + E_N(NVar(X)) \\ &= E^2(X)Var(N) + Var(X)E(N) \end{aligned} \quad (4)$$

From Eq.(4), it can be easily seen that the variance of total earthquake damage is equal to the sum of second moment of earthquake damage amount times variance of the number of earthquakes and variance of earthquake damage amount times expected value of the number of earthquakes.

For more information about collective risk models for compound Poisson distribution, see (Bowers et al. (1997), Gültekin and Erdemir (2010), Kaas et al. (2008).

Premium calculations play an important role in the evaluation of actuarial risk. Traditional premium principle, can be calculated as the expected value principle (EVP) and the standard deviation principle (SDP) as follows, respectively (Hardy, 2006);

$$P = (1 + \theta)E(S) \quad (5)$$

$$P = E(S) + \theta Var(S)^{1/2} \quad (6)$$

According to the premium principle, the premium is bigger than the expected loss (Hardy, 2006). By using the premium loadings, $\theta = 0.10, 0.20, \dots, 0.90, 1.00$, the

variability of the loss is examined for the standard deviation principle.

RESULTS AND DISCUSSION

In this study, we used the dataset of destructive earthquakes in the world having magnitude 7.5 or greater and earthquake damage amount approximately \$1 million or more from National Geophysical Data Center between 1980 and 2014 annually (<https://www.ngdc.noaa.gov/>). Data analysis was done by using IBM SPSS 21.0 and Easy Fit V.5.5 programmes.

In the first step, by using Kolmogorov–Smirnov (K-S) goodness-of-fit test, the distribution of the number of earthquakes comes from Poisson distribution with *K-S* test statistics value 1.143 and related *p*-value 0.146 at $\alpha = 0.05$ significance level. The distribution of the amount of earthquakes comes from log-normal distribution with *K-S* test statistics value 0.11832 and related *p*-value 0.75136 at $\alpha = 0.05$ significance level.

Table 1. Kolmogorov-Smirnov (K-S) goodness-of-fit tests results for the distribution of the number of earthquakes and the amount of earthquakes damages between 1980 and 2014 in the world

	Distribution	K-S test statistics values	Significance values
Number of earthquakes	Poisson	1.143	0.146
Amount of earthquakes damage(\$)	Log-Normal	0.11832	0.75136

Probability density function of the amount of earthquakes between 1980 and 2014 in the world comes from log-normal distribution with parameters $\mu = 15.854$ and $\sigma^2 = 2.1904$.

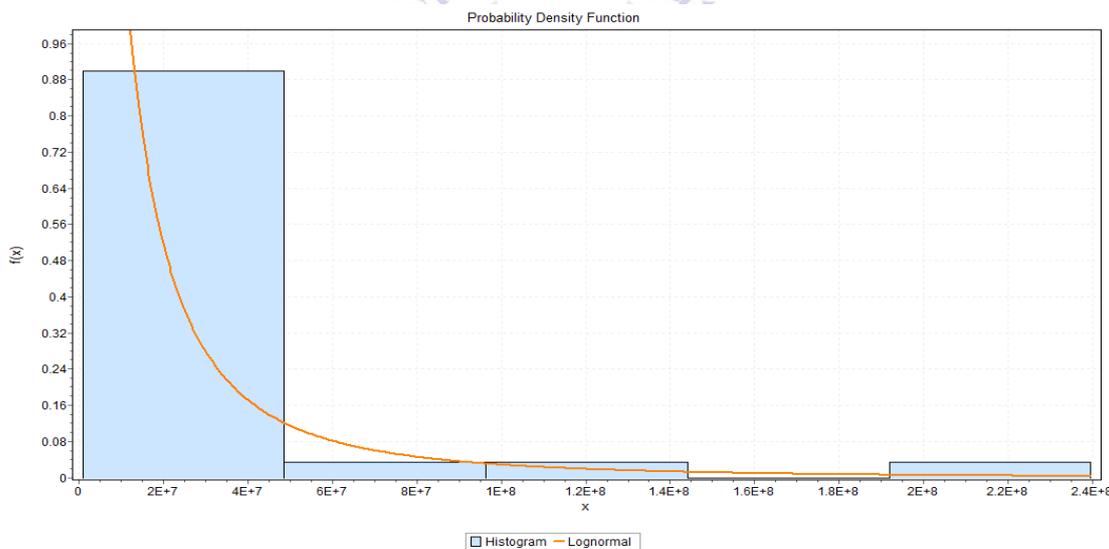


Figure 1. Probability density function of the amount of earthquakes between 1980 and 2014 in the world

In the second step, the expected values and variances of the number of earthquakes and the amount of the earthquake damages between 1980 and 2014 in the world are calculated and given in Table 2 by using Eq.(2), Eq.(3), and Eq.(4).

Table 2. Expected values and variances of the number of earthquakes and the amount of earthquakes damages between 1980 and 2014

Components of <i>S</i>	Distribution	Expected Value	Variance
Number of earthquakes	Poisson	$E(N) = 1.5667$	$Var(N) = 1.5667$
Amount of earthquakes damage(\$)	Log-Normal	$E(X) = 22961925.86$	$Var(X) = 527250039.39$

By using the expected values and variances given in Table 2, expected value and variance of the total earthquake damage amount distribution are as follows;

$$E(S) = 35973683.853$$

$$Var(S) = 1652050123.428$$

For various risk loading factors $\theta = 0.10, 0.20, \dots, 0.90, 1.00$, and by using the EVP and SDP given by Eq.(5), and Eq.(6), premiums are calculated in Table 3.

Table 3. Premiums for various risk loading factors, by using the expected value principle (EVP) and the standard deviation principle (SDP)

Loading factor	Expected Value Principle	Standard Deviation Principle
0,1	39571052.24	40038225.8
0,2	43168420.62	44102767.75
0,3	46765789.01	48167309.69
0,4	50363157.4	52231851.64
0,5	53960525.78	56296393.59
0,6	57557894.17	60360935.53
0,7	61155262.55	64425477.48
0,8	64752630.94	68490019.43
0,9	68349999.32	72554561.37
1,0	71947367.71	76619103.32

From Table 3, by using EVP, the minimum and maximum total amount of earthquake damages in the world are found as 39571052.24 \$, and 39571052.24 \$, respectively. Furthermore by using SDP, the minimum and maximum total amount of earthquake damages in the world are found as 40038225.8 \$, and 76619103.32 \$, respectively.

CONCLUSION

Developing strategies for the natural disasters are possible by using risk assessment measurements tools effectively. Today's technology cannot prevent natural disasters such as earthquakes and it is insufficient to detect time of earthquakes. Measuring the earthquake occurrence risk and taking precautions against various risk levels are necessary for the societies, which are exposed to the natural disasters. In this study, collective risk model (CRM) approach is chosen for the risk assessment of earthquakes occurred between 1980 and 2014 with a magnitude of 7.5 or greater and damage amount approximately \$1 million or more. The total amount of damage is calculated after the distribution of the number of earthquakes is determined to be Poisson, and the amount of damage is log-normal. The results are evaluated for various risk loading factors by using the premium principle method based on the expected value and the standard deviation

principles. From Table 3, it is obviously seen that for small risk loading factors such as 0.1 and 0.2, the minimum total amount of earthquake damages in the world are determined by using EVP and SDP principles. As the risk loading factors increase, total amount of earthquake damages in the world in magnitude 7.5 or greater are beginning to increase significantly.

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