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Theoretical Study of Position-Dependent Mass Particle in Cosmic String Space-Time and External Magnetic Field

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ABSTRACT

In this work, we discuss the case of a non-relativistic quantum particle in the cosmic string space-time under the influence of the presence of external magnetic and exponential potential. It's shown with the help of the Laplace transform method the second-order differential equation of the Schrodinger equation is solvable where it is reduced to a first-order differential equation. We obtain the eigenfunction and the non-relativistic energy spectrum. In addition, these results may be useful in the future study of thermodynamic, optic, and magnetic properties of the atomic interaction of a system particle.

Keywords: Schrodinger equation; Laplace transform method; Position dependent mass; External magnetic field; Cosmic string

INTRODUCTION

The study of the quantum dynamics of a single and diatomic particle interacting with topological defects has attracted the attention of the physics community in recent years, due to its usefulness in many physics areas (Cunha & Silva, 2021; C. O. Edet et al., 2022; Mustafa, 2022). The defect of the topology formed as a result of sudden disintegration and vacuum phase transition in the early universe such as cosmic string, monopoles, domain wall, and texture (Vanchurin et al., 2005). Several aspects of physics for instance cosmology, gauge theory, and condensed matter can be improved with the theory of these defects. Moreover, some study has revealed the energy spectra of relativistic and non-relativistic quantum mechanical systems has been affected by such defects (Rebouças & Tiomno, 1933). Godel (Gödel, 1949) introduces a spacetime metric with an embedded cosmic string that allows the analytical study of physical and mathematical systems in the rotating matter. Reboucas and Tiomno (Rebouças & Tiomno, 1933) studied the spacetime homogeneity and revealed that Godel spacetime is homogeneous. The motion of a quantum particle in a spinning cosmic string space-time also have been examined by Hassanabadi (Hassanabadi et al., 2015) and it is observed that there is a shift in the energy levels of a particle contrary to case static spacetime.

Lately, researchers have devoted their effort to work on the behaviour of particles subjected to the influence of cosmic string topological defects by solving the Schrödinger equation with different potential models and external factors. These interesting works involve electromagnetic potential (Hassanabadi et al., 2015), Kratzer and Morse potentials (Marques & Bezerra, 2005), generalized Morse potential (Nwabuzor et al., 2021a), Yukawa potential (C.

O. Edet et al., 2022), and Mobius square plus Screened Kratzer potential (Okon et al., 2021). The energy spectra investigation of the hydrogen atom in the cosmic string space-time also has been reported (Hassanabadi & Afshardoost, 2015; Ikot et al., 2016; Sobhani et al., 2018).

The study of the energy level of a non-relativistic quantum system is still an interesting issue since it can reveal the behaviour and the physical properties of semiconductor or diatomic molecules (C. Edet & Ikot, 2022; Faniandari et al., 2022; Nwabuzor et al., 2021b; Okorie, Ikot, & Chukwuocha, 2020). Those results can be applied in the study of low-dimensional structures and are important in physics, chemistry, or engineering. The study of the physical properties of the system can be done directly with the partition function that involves the energy eigenvalue equation to the statistical physics. It can be used to interpret and predict mainly thermodynamic properties from the systems which are made up of many particles as well as several phenomena of matter, through the statistical average of dynamic amounts over a specific number of particles (Valencia-Ortega & Arias-Hernandez, 2018). Regarding the problem that arises from the semiconductors heterostructures to organic semiconductors and crystalline solids subject in material science, this solution may be used to model and solve as well (Ahmad El-Nabulsi, 2020). The properties of a molecule also have great relevance in the industrial sector such as playing relevant functions in the synthesis, adsorption, and phase transition of material (Ikot et al., 2019; Jiang et al., 2019a, 2019b; Okorie, Ikot, Chukwuocha, et al., 2020; Song et al., 2017).

The solution of the Schrodinger equation as the dynamic equation which contains as close as possible the real information on the evolution of the above systems (Jia et al., 2017) could be more crucial combined with the position dependent mass particle (PDM) and the influence of an external magnetic field. In a complex environment, the solution of the Schrodinger equation with PDM allows the identification of quantum wave functions (Dong et al., 2022) and the investigation of vibrational properties for diatomic molecules in quantum chemical calculations (Ovando et al., 2019). For this case, we use the exponential type potential to describe the interaction of the RbH, NI, and ScI molecule since it can model the internuclear interaction potential energy of the molecule (Fu et al., 2019).

Several studies have been published regarding the effects of electric or magnetic fields that take into account (Devillanova & Tintarev, 2020; Karayer, 2020; Oliveira & Schmidt, 2020; Purohit et al., 2021). Faniandari et. al reported the solution of the Schrodinger equation with PDM under the influence of an external magnetic field and AB field in 2022 (Faniandari et al., 2022). The analysis of particles under these conditions would be convenient for applications in the fields of condensed matter physics and material science (Eshghi et al., 2017; Khordad & Vaseghi, 2019).

The Laplace transform method was introduced by Pierre-Simon Laplace to simplify the solution of differential equations that contain physical processes (Systems of Units. Some Important Conversion Factors, n.d.), for this case the Schrodinger equation with exponential potential under the influence of an external magnetic field. This method was applied in quantum mechanics by Schrodinger to discuss the radial eigenfunction of the hydrogen atom (Rajbongshi & Singh, 2015). Among other methods that have been applied to solve the Schrodinger equation such as the supersymmetry approach (Faniandari et al., 2020; Suparmi et al., 2020), the factorization method (Jafari et al., 2019; Okorie et al., 2018; Onyenegecha et al., 2020), the Nikiforov-Uvarov method (Karayer, 2020; Karayer & Demirhan, 2021; Okon et al., 2021), the asymptotic iteration method (Ciftci & Kisoglu, 2017; C. O. Edet et al., 2020), and power series expansion method (Khawaja et al., 2018; Korpinar, 2019), the Laplace transform method considered as one of the most effective tools that can reduce the second order differential equation to a first order differential equation conveniently. This method was capable to be used in the field of engineering and physics (Arda & Sever, 2011). Several studies have been reported to use the Laplace transform method to obtain the solution of a relativistic system with a Schrodinger-like equation, involving the Klein-Gordon equation (Miraboutalebi, 2020; Momtazi et al., 2014; Saifullah et al., 2021) and the Dirac equation (Biswas et al., 2016; Eshghi et al., 2012).

LITERATURE REVIEW & METHOD

In the cylindrical coordinate, we have the transformation of variable as follows

$$x = \rho cos\varphi;\ y = \rho sin\varphi;\ z = z \tag{1}$$

and the scaling factor as

$$h_1^2 = h_\rho^2 = \left(\frac{\partial x}{\partial \rho}\right)^2 + \left(\frac{\partial y}{\partial \rho}\right)^2 = cos^2\varphi + sin^2\varphi = 1 \tag{2}$$

$$h_2^2 = h_\varphi^2 = \left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2 = (-\rho sin\varphi)^2 + (\rho cos\varphi)^2 = \rho^2 \tag{3}$$

$$h_3^2 = h_z^2 = \left(\frac{\partial z}{\partial z}\right)^2 = 1 \tag{4}$$

Then $h = h_1 h_2 h_3 = \rho$,

$$\begin{aligned}\nabla_D^2 &= \frac{1}{h}\sum_{j=0}^{D-1}\frac{\partial}{\partial\theta_j}\left(\frac{h}{h_j^2}\frac{\partial}{\partial\theta_j}\right)\\ &= \frac{1}{\rho}\left(\frac{\partial}{\partial\rho}\left(\frac{\rho}{1}\frac{\partial}{\partial\rho}\right)+\frac{\partial}{\partial\varphi}\left(\frac{\rho}{\rho^2}\frac{\partial}{\partial\varphi}\right)+\frac{\partial}{\partial z}\left(\frac{\rho}{1}\frac{\partial}{\partial z}\right)\right)\\ &= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial}{\partial\varphi}\left(\frac{\partial}{\partial\varphi}\right)+\frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}\right)\end{aligned} \tag{5}$$

In general, if we have a D-dimensional hyperspherical coordinate defined as

$$x_1 = r\cos\theta_1 \sin\theta_2 ... \sin\theta_{D-1} \tag{6a}$$

$$x_2 = r\sin\theta_1 \sin\theta_2 ... \sin\theta_{D-1} \tag{6b}$$

$$x_b = r\cos\theta_{b-1} \sin\theta_b ... \sin\theta_{D-1} \tag{6c}$$

$$x_{D-1} = r\cos\theta_{D-2} \sin\theta_{D-1} \tag{6d}$$

$$x_D = r\cos\theta_{D-1} \tag{6e}$$

then the D-dimensional Laplacian in polar coordinates is defined as

$$\nabla_D^2 = \frac{1}{h}\sum_{j=0}^{D-1}\frac{\partial}{\partial\theta_j}\left(\frac{h}{h_j^2}\frac{\partial}{\partial\theta_j}\right) \tag{7a}$$

$$\theta_0 = r \tag{7b}$$

$$h = \prod_{j=0}^{D-1} h_j \tag{7c}$$

$$h_j^2 = \sum_{i=1}^{D}\left(\frac{\partial x_i}{\partial\theta_j}\right)^2 \tag{7d}$$

For cosmic string, there is declination α in φ given by

$$ds^2 = -c^2dt^2 + d\rho^2 + \alpha^2\rho^2 d\varphi^2 + dz^2 \tag{8}$$

therefore $\alpha^2\rho^2 = h_\varphi^2$, and the square of time independent of infinitesimal length ds,

$$ds^2 = d\rho^2 + \alpha^2\rho^2 d\varphi^2 + dz^2 \tag{9}$$

and then we get

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\right) + \frac{1}{(\alpha\rho)^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2} \tag{10}$$

$$\nabla = \left(\hat{\rho}_0\frac{d}{d\rho} + \frac{\hat{\varphi}_0}{\alpha\rho}\frac{d}{d\varphi} + \hat{z}_0\frac{d}{dz}\right) \tag{11}$$

The Schrodinger equation with magnetic field vector which is given as

$$\vec{A} = \left(\frac{Be^{-\sigma\rho}}{2\alpha\rho} + \frac{\Phi_{AB}}{2\pi\rho}\right)\hat{\varphi}_0 \quad (12)$$

$$\tilde{V}(\rho) = \frac{e^{-\sigma\rho}}{\rho} \quad (13)$$

and its position dependent mass that is a function of radial function is given as

$$M(\rho) = \frac{M_0 e^{-\sigma\rho}}{\rho^2} \quad (14)$$

is written as

$$\left\{\left(\bar{p} + \frac{e}{c}\bar{A}\right).\frac{1}{2M}\left(\bar{p} + \frac{e}{c}\bar{A}\right) + (V - E)\right\} f(r,\varphi) = 0 \quad (15)$$

where

$$\bar{p} = -i\nabla = -\mathrm{i}\left(\hat{\rho}_0 \frac{d}{d\rho} + \frac{\hat{\varphi}_0}{\alpha\rho}\frac{d}{d\varphi} + \hat{z}_0\frac{d}{dz}\right) \quad (16)$$

So that

$$\left\{\bar{p}.\frac{1}{2M}\bar{p} + \frac{e}{c}\bar{A}.\frac{1}{2M}\bar{p} + \bar{p}.\frac{1}{2M}\frac{e}{c}\bar{A} + \frac{e}{c}\bar{A}.\frac{1}{2M}\frac{e}{c}\bar{A} + (V - E)\right\} f(r,\varphi) = 0 \quad (17)$$

with

$$\bar{p}.\frac{1}{2M}\bar{p} = \hbar^2 \frac{1}{2}\frac{M'_\rho}{M_\rho^2}\frac{d}{d\rho} - \hbar^2\left(\frac{1}{2M_\rho}\right)\left(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} + \frac{1}{\alpha^2\rho^2}\frac{d^2}{d\varphi^2} + \frac{d^2}{dz^2}\right) \quad (18)$$

$$\bar{p}.\frac{1}{2M}\frac{e}{c}\bar{A} = -i\hbar\frac{e}{c}\frac{1}{2M(\rho)}\left(\left(\frac{B\tilde{V}(\rho)}{2\alpha\rho} + \frac{\Phi_{AB}}{2\pi\rho^2}\right)\right)\frac{d}{\alpha d\varphi} \quad (19)$$

$$\frac{1}{2M(\rho)}\frac{e}{c}\bar{A}.\bar{p} = -i\hbar\frac{e}{c}\frac{1}{2M(\rho)}\left(\left(\frac{B\tilde{V}(\rho)}{2\alpha\rho} + \frac{\Phi_{AB}}{2\pi\rho^2}\right)\right)\frac{d}{\alpha d\varphi} \quad (20)$$

By setting $f(\rho,\varphi,z) = R(\rho)e^{im\varphi}$(9) and inserting equations (10-14, 16-20) into equation (15), we obtain

$$\left\{\hbar^2 \frac{M'_\rho}{M(\rho)}\frac{dR(\rho)}{d\rho} - \hbar^2\left(\left(\frac{d^2R(\rho)}{d\rho^2} + \frac{1}{\rho}\frac{dR(\rho)}{d\rho}\right) - m^2\frac{R}{\alpha^2\rho^2}\right) + 2\hbar\frac{e}{c}\left(\left(\frac{B\tilde{V}(\rho)}{2\alpha\rho} + \frac{\Phi_{AB}}{2\pi\rho^2}\right)\right)\frac{m}{\alpha}R + \left[\left(\frac{e}{c}\left(\frac{B\tilde{V}(\rho)}{2\alpha} + \frac{\Phi_{AB}}{2\pi\rho}\right)\right)^2 R(\rho) + 2M(\rho)(V(\rho) - E)R(\rho)\right]\right\} = 0 \quad (21)$$

$$\frac{M'_\rho}{M(\rho)}\frac{dR(\rho)}{d\rho} - \left(\left(\frac{d^2R(\rho)}{d\rho^2} + \frac{1}{\rho}\frac{dR(\rho)}{d\rho}\right) - m^2\frac{R}{\alpha^2\rho^2}\right) + \frac{2e}{\hbar c}\left(\left(\frac{B\tilde{V}(\rho)}{2\alpha\rho} + \frac{\Phi_{AB}}{2\pi\rho^2}\right)\right)\frac{m}{\alpha}R + \left[\left(\frac{e}{\hbar c}\left(\frac{B\tilde{V}(\rho)}{2\alpha} + \frac{\Phi_{AB}}{2\pi\rho}\right)\right)^2 R(\rho) + \frac{2M}{\hbar^2}(\rho)(V(\rho) - E)R(\rho)\right] = 0 \quad (22)$$

By setting

$$\frac{B\tilde{V}(\rho)}{2\alpha} = \frac{Be^{-\sigma\rho}}{2\alpha\rho},\ \frac{\phi_{AB}}{\phi_o} = \xi,\ \frac{eB}{M_0 c} = \omega_c, \frac{\hbar c}{e} = \frac{hc}{2\pi e} = \frac{\phi_0}{2\pi} \quad (23)$$

then the Schrodinger in equation (22) becomes

$$\frac{M'_\rho}{M(\rho)}\frac{dR(\rho)}{d\rho} - \left(\frac{d^2R(\rho)}{d\rho^2} + \frac{1}{\rho}\frac{dR(\rho)}{d\rho}\right) + 2\frac{M_0}{\hbar}\omega_c\frac{\tilde{V}(\rho)}{2\alpha\rho}\left(\frac{m}{\alpha} + \xi\right)R + \frac{1}{\rho^2}\left(\xi + \frac{m}{\alpha}\right)^2 R + \left(\frac{M_0}{\hbar}\omega_c\frac{\tilde{V}(\rho)}{2\alpha}\right)^2 R + \frac{2M}{\hbar^2}(\rho)(V(\rho) - E)R(\rho) = 0 \quad (24)$$

By using equations (12-13) and (24) we have

$$-\left(\frac{d^2R(\rho)}{d\rho^2} + \left(\frac{1}{\rho} - \frac{M'_\rho}{M(\rho)}\right)\frac{dR(\rho)}{d\rho}\right) + 2\frac{M_0}{\hbar}\omega_c\frac{1}{2\alpha\rho}\frac{V_0 e^{-\sigma\rho}}{\rho}\left(\frac{m}{\alpha} + \xi\right)R + \frac{1}{\rho^2}\left(\xi + \frac{m}{\alpha}\right)^2 R + \left(\frac{M_0}{\hbar}\omega_c\frac{V_0 e^{-\sigma\rho}}{2\alpha\rho}\right)^2 R + \frac{2M}{\hbar^2}(\rho)(V(\rho) - E)R(\rho) = 0 \quad (25)$$

with

$$M(\rho) = \frac{M_0 e^{-\sigma\rho}}{\rho^2};\ M'(\rho) = \frac{M_0 e^{-\sigma\rho}}{\rho^2}\left(-\sigma - \frac{2}{\rho}\right) \quad (26)$$

$$\frac{M'_\rho}{M_\rho} = -\sigma - \frac{2}{\rho} \quad (27)$$

For small σ, we have approximate values for

$$V(\rho) = V_1 \tanh \sigma\rho \approx V_1 \sigma\rho \quad (28)$$

$$\frac{M_0 e^{-\sigma\rho}}{\rho^2} \approx \frac{M_0(1-\sigma\rho)}{\rho^2} \approx M_0\left(\frac{1}{\rho^2} - \frac{\sigma}{\rho}\right) \quad (29)$$

$$\tilde{V}(\rho) = \frac{V_0 e^{-\sigma\rho}}{\rho} \approx V_0 \frac{(1-\sigma\rho)}{\rho} \approx V_0\left(\frac{1}{\rho} - \sigma\right) \quad (30)$$

By applying equations (25-30) we have

$$-\left(\frac{d^2R(\rho)}{d\rho^2} + \left(\frac{3}{\rho} + \sigma\right)\frac{dR(\rho)}{d\rho}\right) + 2\frac{M_0}{\hbar}\omega_c \frac{V_0\left(\frac{1}{\rho} - \sigma\right)}{2\alpha\rho}\left(\frac{m}{\alpha} + \xi\right)R + \frac{1}{\rho^2}\left(\xi + \frac{m}{\alpha}\right)^2 R + \left(\frac{M_0}{\hbar}\omega_c \frac{V_0\left(\frac{1}{\rho} - \sigma\right)}{2\alpha}\right)^2 R + \frac{2M_0\left(\frac{1}{\rho^2} - \frac{\sigma}{\rho}\right)}{\hbar^2}(V_1\sigma\rho - E)R(\rho) = 0 \quad (31)$$

By simple mathematical manipulation equation (31) we have

$$-\left\{\frac{d^2R(\rho)}{d\rho^2} + \left(\frac{3}{\rho} + \sigma\right)\frac{\partial R(\rho)}{\partial\rho}\right\} - \frac{1}{\rho}\left\{2\sigma\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\left[\left(\frac{m}{\alpha} + \xi\right) + \left(\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\right)\right] - \frac{2M_0\sigma}{\hbar^2}(V_1 + E)\right\}R + \frac{1}{\rho^2}\left[\left\{\left(\xi + \frac{m}{\alpha}\right) + \left(\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\right)\right\}^2 - \frac{2M_0}{\hbar^2}E\right]R + \left\{\left(\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\sigma\right)^2 - \frac{2M_0}{\hbar^2}V_1\sigma^2\right\}R(\rho) = 0 \quad (32)$$

By setting

$$\left\{2\sigma\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\left[\left(\frac{m}{\alpha} + \xi\right) + \left(\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\right)\right] - \frac{2M_0\sigma}{\hbar^2}(V_1 + E)\right\} = \delta \quad (33)$$

$$\left\{\left(\xi + \frac{m}{\alpha}\right) + \left(\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\right)\right\}^2 - \frac{2M_0}{\hbar^2}E = \tau \quad (34)$$

in equation (32), we have

$$-\left\{\frac{d^2R(\rho)}{d\rho^2} + \left(\frac{3}{\rho} + \sigma\right)\frac{\partial R(\rho)}{\partial\rho}\right\} - \frac{\delta}{\rho}R + \frac{\tau}{\rho^2}R + \left\{\left(\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\sigma\right)^2 - \frac{2M_0}{\hbar^2}V_1\sigma^2\right\}R(\rho) = 0 \quad (35)$$

Equation (35) is solved by setting

$$R(\rho) = \frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{3}{2}}}Q \quad (36)$$

$$\frac{\partial R(\rho)}{\partial\rho} = -\frac{\sigma}{2}\frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{3}{2}}}Q - \frac{3e^{-\frac{\sigma\rho}{2}}}{2\rho^{\frac{5}{2}}}Q + \frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{3}{2}}}Q' \quad (37)$$

$$\frac{\partial^2 R(\rho)}{\partial\rho^2} = \frac{\sigma^2}{4}\frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{3}{2}}}Q + 2\frac{3\sigma}{4}\frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{5}{2}}}Q - 2\frac{\sigma}{2}\frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{3}{2}}}Q' + \frac{5.3e^{-\frac{\sigma\rho}{2}}}{4\rho^{\frac{7}{2}}}Q - 2\frac{3e^{-\frac{\sigma\rho}{2}}}{2\rho^{\frac{5}{2}}}Q' + \frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{3}{2}}}Q'' \quad (38)$$

$$\left(\frac{3}{\rho} + \sigma\right)\frac{\partial R(\rho)}{\partial\rho} = \left(-\frac{3}{\rho}\frac{\sigma}{2}\frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{3}{2}}}Q - \frac{3}{\rho}\frac{3e^{-\frac{\sigma\rho}{2}}}{2\rho^{\frac{5}{2}}}Q + \frac{3}{\rho}\frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{3}{2}}}Q'\right) + \left(-\sigma\frac{\sigma}{2}\frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{3}{2}}}Q - \sigma\frac{3e^{-\frac{\sigma\rho}{2}}}{2\rho^{\frac{5}{2}}}Q + \sigma\frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{3}{2}}}Q'\right) \quad (39)$$

From equations (38-39) we obtain

$$\frac{d^2R(\rho)}{d\rho^2} + \left(\frac{3}{\rho} + \sigma\right)\frac{\partial R(\rho)}{\partial\rho} = -\frac{\sigma^2}{4}\frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{3}{2}}}Q - \frac{3\sigma}{2}\frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{5}{2}}}Q - \frac{3e^{-\frac{\sigma\rho}{2}}}{4\rho^{\frac{7}{2}}}Q + \frac{e^{-\frac{\sigma\rho}{2}}}{\rho^{\frac{3}{2}}}Q'' \quad (40)$$

By substituting equations (36, 40) into equation (35) we obtain

$$Q'' - \left(\frac{\frac{3}{2}\sigma - \delta}{\rho}\right)Q - \frac{\frac{3}{4} + \tau}{\rho^2}Q - \left\{\left(\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\right)^2 + \frac{1}{4} - \frac{2M_0}{\hbar^2}V_1\right\}\sigma^2 Q = 0 \quad (41)$$

By setting

$$\left\{\left(\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\right)^2 + \frac{1}{4} - \frac{2M_0}{\hbar^2}V_1\right\}\sigma^2 = \varepsilon^2;\ \frac{3}{2}\sigma - \delta = v^2;\ \ \frac{3}{4} + \tau = \gamma^2 \quad (42)$$

in equation (41) then it reduces to

$$Q'' - \left(\frac{v^2}{\rho}\right)Q - \frac{\gamma^2}{\rho^2}Q - \varepsilon^2 Q \ = 0 \tag{43}$$

RESULTS & DISCUSSION

The second order differential of the Schrodinger equation in (43) is solved using Laplace transform method. The first step is by setting $Q = \sqrt{\rho}\phi$. So that

$$Q' = \sqrt{\rho}\phi' + \frac{1}{2}\frac{\phi}{\sqrt{\rho}} \tag{44}$$

$$Q'' = \sqrt{\rho}\phi'' + \frac{1}{2}\frac{\phi\prime}{\sqrt{\rho}} + \frac{1}{2}\frac{\phi\prime}{\sqrt{\rho}} - \frac{1}{4}\frac{\phi}{\rho\sqrt{\rho}} \tag{45}$$

Equations (33-45) are inserted into equation (43) and we obtain

$$\rho^2\phi'' + \rho\phi' - \rho v^2\phi - \kappa^2\phi - \rho^2\varepsilon^2\phi \ = 0 \tag{46}$$

By arranging

$$\phi = \rho^\eta f(\rho) \tag{47}$$

Equation (47) is inserted into equation (46), so we can define

$$\rho^2 f''(\rho) + (2\eta+1)\rho f'(\rho) + \eta^2 f(\rho) - (\rho v^2 + \kappa^2 + \rho^2\varepsilon^2)f(\rho) \ = 0 \tag{48}$$

Then, equation (48) reduces by setting $\eta = -\kappa$ into

$$\rho f''(\rho) - (2\kappa - 1)f'(\rho) - (v^2 + \rho\varepsilon^2)f(\rho) \ = 0 \tag{49}$$

Arranging $\kappa = a_1$; $\varepsilon^2 \to \varepsilon_n^2$; and $v^2 = a_2^2$, so equation (49) become

$$\rho f''(\rho) - (2a_1 - 1)f'(\rho) - (a_2^2 + \rho\varepsilon_n^2)f(\rho) \ = 0 \tag{50}$$

By applying Laplace Transform in equation (50) as follows

$$\mathcal{L}(f(\rho)) = F(t) = \int_0^\infty f(\rho)e^{-t\rho}dt \tag{51}$$

$$\mathcal{L}(\rho f''(\rho)) = -2tF(t) - t^2\frac{dF}{dt} \tag{52}$$

$$\mathcal{L}(\rho f'(\rho)) = -tF'(t) - F(t) \tag{53}$$

$$\mathcal{L}(f'(\rho)) = tF(t) \tag{54}$$

$$\mathcal{L}(\rho f(\rho)) = -F'(t) \tag{55}$$

then equation (50) becomes

$$(t^2 - \varepsilon_n^2)\frac{dF}{dt} + \{(2a_1+1)t + a_2^2\}F(t) = 0 \tag{56}$$

The identity is taken by

$$\frac{1}{(t+\varepsilon_n)(t-\varepsilon_n)} = \frac{1}{2\varepsilon_n}\left(\frac{1}{t-\varepsilon_n} - \frac{1}{t+\varepsilon_n}\right) \tag{57}$$

in equation (56) then we determine

$$\frac{dF}{F} = -\frac{\{(2a_1+1)t + a_2^2\}}{(t^2 - \varepsilon_n^2)}dt = \frac{1}{2\varepsilon_n}\left\{-\frac{\{(2a_1+1)t + a_2^2\}}{t-\varepsilon_n}dt + \frac{\{(2a_1+1)t + a_2^2\}}{t+\varepsilon_n}dt\right\} \tag{58}$$

By integrating equation (58) we have the wave function as

$$F = \left\{\frac{(t-\varepsilon_n)}{(t+\varepsilon_n)}\right\}^{\frac{-(2a_1+1)}{2} - \frac{a_2^2}{2\varepsilon_n}}(t+\varepsilon_n)^{-(2a_1+1)} \tag{59}$$

In order for the wave function to be single-valued, then from equation (36), we have the condition

$$\frac{-(2a_1+1)}{2} - \frac{a_2^2}{2\varepsilon_n} = n \tag{60}$$

and by considering the condition in equation (60) we expand equation (59) as

$$F = \left\{\frac{(t-\varepsilon_n)}{(t+\varepsilon_n)}\right\}^{\frac{-(2a_1+1)}{2} - \frac{a_2^2}{2\varepsilon_n}}(t+\varepsilon_n)^{-(2a_1+1)} = \sum_{k=0}^{n}\frac{(-1)^k n!}{(n-k)!k!}(2\varepsilon_n)^k(t+\varepsilon_n)^{-(2a_1+1)-k} \tag{61}$$

The Inverse Laplace Transform is applied, then we get $f(\rho)$ as

$$f(\rho) = \rho^{2a_1}e^{-\varepsilon_n\rho}\sum_{k=0}^{n}\frac{(-1)^k n!\,\Gamma(2a_1+1)}{(n-k)!k!\,\Gamma(2a_1+1+k)}(2\varepsilon_n\rho)^k \tag{62}$$

Hence the radial wave function is obtained as

$$f(\rho) = \rho^{2a_1}e^{-\varepsilon_n\rho}{}_1F_1(-n, 2a_1+1; 2\varepsilon_n\rho) \tag{63}$$

where the confluent hypergeometric function is defined as

$$F_1(-n,\sigma;x) = \sum_{p=0}^{n} \frac{(-1)^p n!\,\Gamma(\sigma)}{(n-p)!p!\,\Gamma(\sigma+p)}(x)^p \quad (64)$$

The non-relativistic energy equation is determined from equation (60). Considering the condition of

$$\frac{-(2a_1+1)}{2} = n + \frac{a_2^2}{2\varepsilon_n} \quad (65)$$

where $\kappa = a_1;\ \varepsilon^2 \rightarrow \varepsilon_n^2;\ v^2 = a_2^2;\ \eta = -\kappa$

$$\frac{-(2\kappa+1)}{2} = n + \frac{v^2}{2\varepsilon^2} \quad (66)$$

Then we look into equations (33)-(34) and (42), we substitute the variables $\delta, \tau, \varepsilon^2, v^2, \gamma^2$. So that

$$\frac{-(2\kappa+1)}{2} = n + \frac{\frac{3}{2}\sigma - \left\{2\sigma\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\left[\left(\frac{m}{\alpha}+\xi\right)+\left(\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\right)\right] - \frac{2M_0\sigma}{\hbar^2}(V_1+E)\right\}}{2\left\{\left(\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\right)^2 + \frac{1}{4} - \frac{2M_0}{\hbar^2}V_1\right\}\sigma^2} \quad (67)$$

The non-relativistic energy is obtained as

$$E = \frac{\hbar^2}{2M_0\sigma}\left[\left(n+\frac{2\kappa+1}{2}\right)\left(2\left\{\left(\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\right)^2 + \frac{1}{4} - \frac{2M_0}{\hbar^2}V_1\right\}\sigma^2\right) + \frac{3}{2}\sigma - \left\{2\sigma\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\left[\left(\frac{m}{\alpha}+\xi\right) + \left(\frac{M_0}{\hbar}\omega_c\frac{V_0}{2\alpha}\right)\right]\right\} - V_1\frac{2M_0\sigma}{\hbar^2}\right] \quad (68)$$

In this part, we discuss the theoretical prediction of the energy level for the three selected diatomic molecules with the natural units. The energy of the system in equation (68) can be used to examine the behavior of the particle under the influence of the external magnetic fields. Several parameters can be applied such as for the simple diatomic molecules. From equation (68), the quantum number *n* and the magnetic quantum number *m* are linear to the energy E, so we can infer if the energy increases linearly with the increase of the quantum number. The presence of the external magnetic field also has influenced the energy level to increase, since it has a linear ratio.

The energy eigenvalue tends to have a positive value and was increased linearly with an increase in quantum number. The energy also tends to increase with the increase of σ because of its linear ratio. The particle is more bound with small values of σ, while less bound with the increasing values of α. The molecule's mass change has a non-linear effect on the plot of the energy level. The weigher particle has a smaller energy level for the fixed value of external magnetic field strength *B*. It is obvious that the increasing value of potential parameter V_0 caused the energy increases. The molecule's mass change might have a negative effect on the plot of the energy because the relationship is inversely proportional. The weigher particle has a smaller energy level for the fixed value of external magnetic field strength *B*.

CONCLUSION

To summarize, we have solved the approximate solution of the Schrodinger equation with hyperbolic function position-dependent mass for an exponential potential by using the hypergeometric method. We consider the system influenced by external magnetic forces. The second-order differential equation of the Schrodinger equation is reduced to the first-order differential equation and so the eigenfunction and eigen energy values are obtained. We also analyzed the behavior of the bound state energy levels. It can be infered that the energy value depends on the quantum number, potential parameter, mass of the molecule, and magnetic field strength. These results can motivate the further study of molecular physics for several molecules.

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