

Numerical Analysis of the SIR Model Using the Fourth-Order Runge-Kutta Method

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ABSTRACT

The Susceptible–Infected–Recovered (SIR) model is a fundamental mathematical framework for analyzing the transmission dynamics of infectious diseases. Due to its nonlinear structure, analytical solutions are generally difficult to obtain, making numerical analysis an essential approach. This study presents a numerical analysis of the SIR model using the fourth-order Runge–Kutta (RK4) method, with emphasis on numerical stability, solution accuracy, and epidemiological interpretability. Numerical simulations are conducted and compared with the first-order Euler method to examine differences in solution behavior. The results indicate that the RK4 method produces smoother solution trajectories, more accurate estimation of the infection peak, and better preservation of population consistency than the Euler method. These findings confirm that RK4 is not only computationally superior but also more reliable for interpreting epidemic dynamics, making it suitable for applied and educational epidemiological modeling.

Keywords: SIR model, numerical analysis, Runge–Kutta method, epidemic modeling, numerical simulation

INTRODUCTION

Infectious diseases remain a major concern in public health, particularly during epidemic and pandemic outbreaks.

Understanding the transmission mechanism of infectious diseases is essential for designing effective prevention and control strategies. Mathematical modeling has become an important approach to represent and analyze the spread of diseases in a population.¹

One of the most fundamental mathematical models in epidemiology is the Susceptible–Infected–Recovered (SIR) model.² This model describes the interaction among individuals who are susceptible to infection, currently infected, and recovered with immunity. The SIR model is formulated as a system of nonlinear ordinary differential equations, which makes analytical solutions difficult to obtain in most cases.³

To overcome this limitation, numerical methods are commonly employed to approximate the solutions of the SIR model.^{3,4} Among various numerical approaches, the fourth-order Runge–Kutta (RK4) method is widely recognized for its high accuracy and numerical stability.⁵ Therefore, this study aims to apply the RK4 method to solve the SIR model and to analyze the dynamic behavior of disease transmission based on numerical simulations.

LITERATURE REVIEW

The SIR model was first introduced by Kermack and McKendrick as a fundamental framework for studying epidemic processes and has since been widely applied to various

infectious diseases.⁶ Hethcote further emphasized the role of compartmental models in understanding disease dynamics and supporting public health interventions. Numerical methods play a crucial role in solving epidemiological models formulated as nonlinear differential equations.⁷ Runge–Kutta methods are widely recognized for their accuracy and stability, particularly higher-order schemes. Butcher highlighted the effectiveness of Runge–Kutta methods in numerical analysis, while Kreyszig discussed their broad applications in applied mathematics and engineering.^{8,9} Although previous studies demonstrate the accuracy of higher-order methods such as RK4, explicit numerical analysis focusing on stability, solution consistency, and epidemiological interpretability in the SIR model context remains limited. This gap motivates the present study.¹⁰

MATERIALS & METHODS

SIR Mathematical Model

The SIR model is described by the following system of ordinary differential equations:

$\frac{dS}{dt} = -\beta SI,$	(1)
$\frac{dI}{dt} = \beta SI - \gamma I,$	(2)
$\frac{dR}{dt} = \gamma I.$	(3)

where $S(t)$, $I(t)$, and $R(t)$ represent the proportions of susceptible, infected, and recovered individuals at time t , respectively. The parameter β denotes the disease transmission rate, while γ represents the recovery rate. The total population is assumed to be constant, such that:

$S(t) + I(t) + R(t) = 1.$	(4)
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Numerical Methods

The numerical simulations are performed using two numerical schemes: the first-order Euler method and the fourth-order Runge–Kutta (RK4) method. For a general first-order ordinary differential equation,

$\frac{dy}{dt} = f(t, y),$	(5)
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the RK4 method is defined as follows:

$k_1 = f(t_n, y_n)$	(6)
$k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right),$	(7)
$k_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right),$	(8)
$k_4 = f(t_n + h, y_n + hk_3),$	(9)
$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4),$	(10)

where h denotes the time step size. This numerical scheme is applied simultaneously to each equation in the SIR system.

Simulation Parameters

The numerical simulations are conducted using the following parameter values. The initial conditions are set as $S(0) = 0.99$, $I(0) = 0.01$, and $R(0) = 0$, representing a population where a small proportion of individuals is initially infected. The

transmission rate and recovery rate are chosen as

$\beta = 0.5$ and $\gamma = 0.1$, respectively. A constant time step size of $h = 0.1$ is used to ensure numerical stability and accuracy. The simulation is performed over the time interval $t \in [0,60]$ to capture the complete dynamics of the disease transmission process.

RESULT

This section presents the numerical results of the SIR model obtained using the fourth-order Runge–Kutta (RK4) method and compares them with the results produced by the Euler method. The analysis focuses on the dynamic behavior of the susceptible, infected, and recovered populations as well as the numerical accuracy of each method.

Numerical Results Using RK4

The numerical solutions obtained using the RK4 method are summarized in Table 1.

Table 1. Numerical Solution of the SIR Model Using RK4

Time (t)	$S(t)$	$I(t)$	$R(t)$
0	0.9900	0.0100	0.0000
10	0.7346	0.1812	0.0842
20	0.3128	0.4015	0.2857
30	0.1184	0.2436	0.6380
40	0.0627	0.0921	0.8452
60	0.0512	0.0154	0.9334

The results show that the infected population increases rapidly during the early stage of the epidemic, reaches a peak around $t \approx 20$, and then decreases as recovery dominates the transmission process.

Comparison Between Euler and RK4 Methods

The numerical results obtained using the Euler method are compared with those obtained using the RK4 method, as presented in Table 2.

Table 2. Comparison of Infected Population Obtained by Euler and RK4 Methods

Time (t)	$I(t)$ Euler	$I(t)$ RK4
10	0.1934	0.1812
20	0.4269	0.4015
30	0.2688	0.2436
40	0.1076	0.0921
60	0.0287	0.0154

Table 2 indicates that the Euler method consistently overestimates the infected population, particularly near the epidemic peak, whereas the RK4 method provides more stable and accurate results.

Graphical Representation

The graphical results are presented to visually illustrate the dynamic behavior of the SIR model and to compare the numerical performance of the Euler and fourth-order Runge–Kutta (RK4) methods over the simulation period.

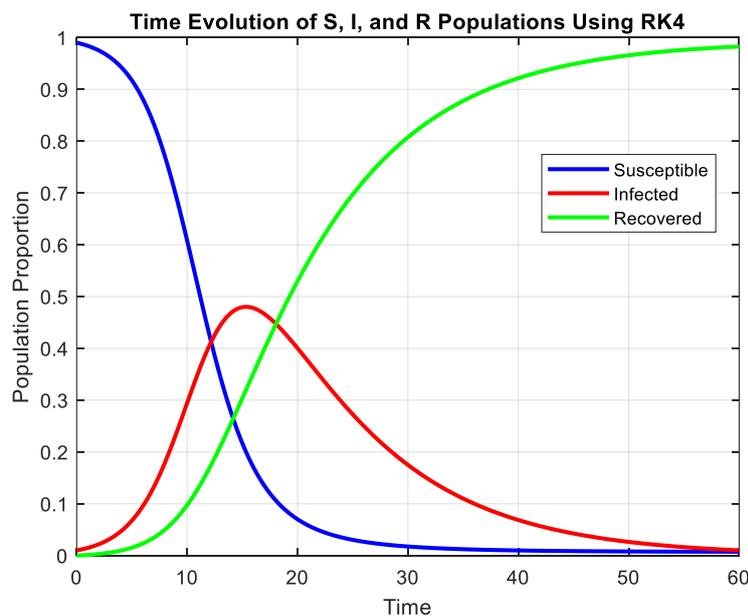


Figure 1. Time Evolution of S, I, and R Populations Using the RK4 Method

Figure 1 illustrates the temporal evolution of the susceptible, infected, and recovered populations obtained using the fourth-order Runge–Kutta (RK4) method. At the initial stage of the simulation, the susceptible population $S(t)$ decreases rapidly as a consequence of frequent interactions between susceptible and infected individuals, reflecting the active transmission phase of the disease. This decrease in $S(t)$ is accompanied by a corresponding increase in the infected population $I(t)$, indicating the early growth of the epidemic.

As time progresses, the infected population continues to increase and reaches a maximum value around the mid-simulation period. This peak represents the point at which the rate of new infections begins to be balanced by the recovery process. After this peak, the infected population declines steadily, which is primarily driven by the increasing number of recovered individuals and the decreasing pool of susceptible individuals. This behavior is consistent with the theoretical properties of the SIR model

and reflects the natural progression of an epidemic toward eventual stabilization.

Meanwhile, the recovered population $R(t)$ increases monotonically throughout the simulation. This monotonic growth indicates that recovered individuals accumulate over time as infected individuals transition into the recovered compartment. The smooth and continuous increase of $R(t)$ further demonstrates the numerical stability of the RK4 method, as no artificial oscillations or numerical artifacts are observed.

Overall, the smooth trajectories of all three compartments indicate that the RK4 method provides a stable and accurate numerical approximation of the SIR model. In addition, the conservation of the total population, implicitly observed through the complementary behavior of $S(t)$, $I(t)$, and $R(t)$, confirms the consistency of the numerical scheme. These results demonstrate that the RK4 method is well suited for capturing the dynamic characteristics of infectious disease transmission and for supporting reliable interpretation of epidemic behavior.

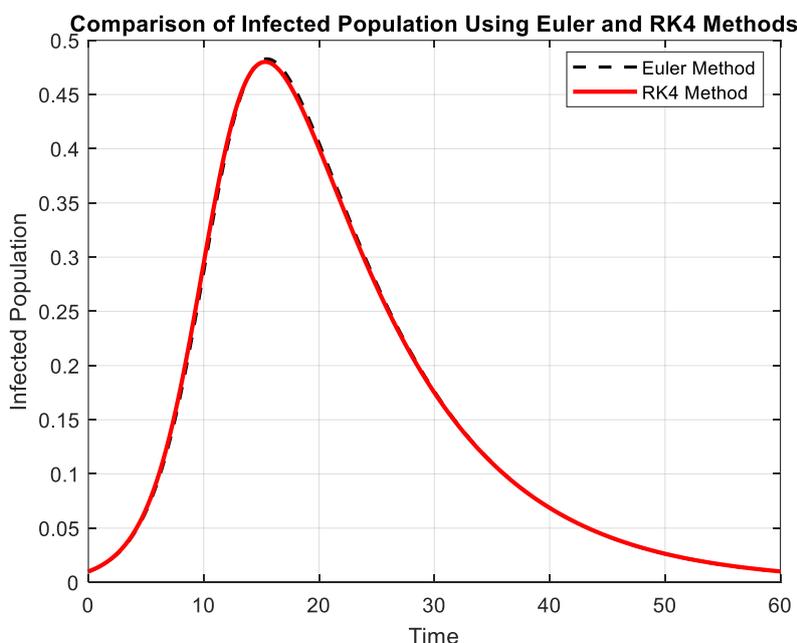


Figure 2. Comparison of Infected Population Using Euler and RK4 Methods

Figure 2 presents a comparison of the infected population obtained using the first-

order Euler method and the fourth-order Runge–Kutta (RK4) method over the same

simulation interval. The results show that both methods capture the general trend of the epidemic, characterized by an initial increase in the number of infected individuals followed by a decline. However, noticeable differences appear in the magnitude and smoothness of the solution curves. The Euler method produces a higher infection peak and a less smooth trajectory, particularly around the peak region, which indicates the presence of larger numerical errors associated with its lower-order approximation.

In contrast, the RK4 method yields a smoother infected population curve with a lower and more realistic peak value. This behavior reflects the higher accuracy of the RK4 method in approximating nonlinear dynamics and reducing local truncation error. The improved performance of RK4 is especially important near the epidemic peak, where numerical inaccuracies can significantly affect interpretation. These results demonstrate that higher-order numerical methods such as RK4 provide more reliable solutions for epidemic modeling, particularly when accurate estimation of infection peaks and overall disease dynamics is required.

DISCUSSION

The numerical simulation results obtained in this study are consistent with the theoretical behavior of the classical SIR epidemic model. As shown in the numerical tables and graphical results, the infected population initially increases rapidly due to active disease transmission, reaches a peak when the transmission and recovery processes are in balance, and subsequently declines as the number of recovered individuals increases and the susceptible population decreases. This pattern confirms that the numerical scheme accurately captures the fundamental dynamics of epidemic spread, including the emergence and decline of infection peaks.

Furthermore, the comparison between the Euler and fourth-order Runge–Kutta (RK4) methods demonstrates the advantages of

higher-order numerical schemes in solving nonlinear epidemiological models. The Euler method tends to overestimate the infected population, particularly near the epidemic peak, due to larger local truncation errors. In contrast, the RK4 method provides smoother solution trajectories, improved numerical stability, and more accurate peak estimation for the same time step size. These properties are essential for reliable interpretation of epidemic dynamics, especially in applied and educational contexts where numerical accuracy directly influences model-based insights. Therefore, the RK4 method is shown to be a robust and appropriate numerical approach for analyzing the SIR model and similar nonlinear epidemic systems.

CONCLUSION

This study confirms that the fourth-order Runge–Kutta (RK4) method is an effective and reliable numerical approach for solving the SIR epidemic model. The numerical results demonstrate that RK4 produces stable and smooth solution trajectories, accurately captures the infection peak, and preserves population consistency when compared with the first-order Euler method. These findings highlight the suitability of RK4 for analyzing nonlinear epidemic dynamics and for supporting clear interpretation of disease transmission behavior. Future studies may extend the present model by incorporating additional compartments, time-dependent parameters, or real epidemiological data to enhance its applicability in more complex and realistic scenarios.

Declaration by Authors

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