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A Cognitive Method for Calculating Squares of Numbers

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ABSTRACT

Squaring of numbers is traditionally taught using standard long multiplication or algebraic expansion identities. While these methods are mathematically rigorous and universally accepted, they may appear cognitively demanding for some learners because they involve simultaneous management of multiplication, carrying operations, symbolic manipulation and larger numbers. This paper presents a method that attempts to reorganize squaring into smaller multiplications and sequential steps. The proposed method emerged empirically by observing square values across decadal intervals and subsequently attests algebraic confirmation. The paper outlines the method and provides algebraic justification for the same. It also shows that, while mathematically equivalent, the new approach emphasizes visual pattern recognition and may reduce cognitive load during calculation, suggesting potential value as a pedagogical supplement. The proposed approach makes the mental computation of squares of numbers easier, making it particularly suitable for students who respond more effectively to pattern-based reasoning than to formal symbolic computation or the memorization of large numerical values.

Keywords: Squaring Method, Cognitive mathematics, Pattern recognition, Pedagogical mathematics, Squares of numbers, Mental Calculation

INTRODUCTION

Squaring numbers is one of the foundational operations in elementary mathematics and serves as an important basis for algebra, geometry and numerical reasoning. In most educational settings, students are introduced to squaring either through standard long multiplication or through algebraic identities such as:

$$\boxed{(a + b)^2 = a^2 + 2ab + b^2}$$

Although these methods are mathematically complete and universally accepted, many learners may experience difficulty performing mental squaring because the procedures require simultaneous handling of multiplication facts, carrying operations, intermediate summations, and symbolic manipulation.

Educational research in mathematical cognition has emphasized the importance of relational understanding and the management of working-memory demands during problem solving. [1-3] This method attempts to reorganize squaring into smaller sequential steps. For many learners, such structured progression provides a cognitively accessible alternative to simultaneous symbolic manipulation and carry management. [4,5]

The proposed method was discovered empirically through observation of differences in square values across decadal intervals. The resulting method uses anchor numbers and finite-difference propagation to reconstruct squares through sequential increments with reduced reliance on large direct multiplications. The approach is in the spirit of classical mental arithmetic methods such as those of Tirthaji [6] and Trachtenberg.[7]

MATERIALS & METHODS

List of Variables, Symbols, and Functions Used in This Method

Below are the variables and symbols appearing in the squaring method along with their description:

- **(N):** Number whose square we are finding (integer).
- **(X):** Tens and higher order component of (N) excluding the unit digit.
- **(Y):** Units digit of (N) (integer 0–9).
- **(A):** Anchor number (integer). A nearby base number chosen for efficiency, typically $A \in \{10X, 10X+5, 10(X+1)\}$.
- **S or S(A):** Tens and higher order component of the square of the anchor number, $S = \lfloor A^2/10 \rfloor$ (integer).
- **(K):** Increment constant (integer).
- **(P):** Prefix of the square (integer) excluding the unit digit.
- **(Q):** Unit digit of the square (integer 0–9).
- **Floor function ⌊·⌋:** Greatest integer function.
- **Mod function:** Used for finding the remainder.

Let N be a two-digit number whose square we are calculating. N is represented as:

$$\boxed{N = 10X + Y \text{ (algebraically)} \mid N = XY \text{ (visually)}}$$

N^2 is represented as:

$$\boxed{N^2 = 10P + Q \text{ (algebraically)} \mid N^2 = PQ \text{ (visually)}}$$

Table 1 and Table 2 display the squares of integers in two successive decadal intervals, together with the prefix P and the first-difference ΔP.

Table 1: Squares of Numbers (20–30)

N	N²	P	ΔP = P(N) - P(N-1) or K
20	400	40	—
21	441	44	4
22	484	48	4
23	529	52	4
24	576	57	5
25	625	62	5
26	676	67	5
27	729	72	5
28	784	78	6
29	841	84	6
30	900	90	6

Table 2: Squares of Numbers (30–40)

N	N²	P	ΔP = P(N) - P(N-1) or K
30	900	90	—
31	961	96	6
32	1024	102	6
33	1089	108	6
34	1156	115	7
35	1225	122	7
36	1296	129	7
37	1369	136	7
38	1444	144	8
39	1521	152	8
40	1600	160	8

Observations from the Tables (Decadal Intervals 20–30 and 30–40)

From the above tables, the following facts are evident:

1. The differences between successive P-values follow a fixed pattern.
2. Within each decadal interval, the P-value differences repeat the particular pattern as:

$$\boxed{K, K, K, K+1, K+1, K+1, K+1, K+2, K+2, K+2}$$

3. Since the corresponding values of 'X' for the above decadal intervals 20–30 & 30–40 are 2 & 3 respectively, the table suggests the relation:

$$\boxed{K = 2X}$$

This conclusion is reached purely from numerical observation. We now derive 'P' and 'Q' separately to get the resulting square.

The Step-Wise Squaring Method Based on Empirical Observation

The empirical observations above naturally lead to the following steps:

Step 1: Unit Digit (Q)
The unit digit of the resultant square is obtained immediately using the Modulo 10 property as:

$$Q(N) \equiv Y^2 \pmod{10} \quad (1)$$

Step 2: Anchor Number Selection
Select a nearby anchor number 'A' whose square is very easy to compute mentally. Common choices are:

$$A \in \{10X,\ 10X+5,\ 10(X+1)\} \quad (2)$$

To calculate P of the squared number (N^2), travel forward or backward in incremental steps (K, K+1, K+2, or combination) from the selected anchor number. Define:

$$S(A) = \lfloor A^2 / 10 \rfloor \quad (3)$$

Here ⌊x⌋ denotes the greatest integer function (floor function).

Step 3: Increment Constant
From the table:

$$K = 2X \quad (4)$$

Step 4: Finite-Difference Propagation to Calculate (P)

Starting from S(A), move stepwise from the anchor number by adding or subtracting successive increments to reach the target number N (whose square we are calculating) to calculate P(N):

$$P(N) = S(A) \pm (\text{incremental steps}) \quad (5)$$

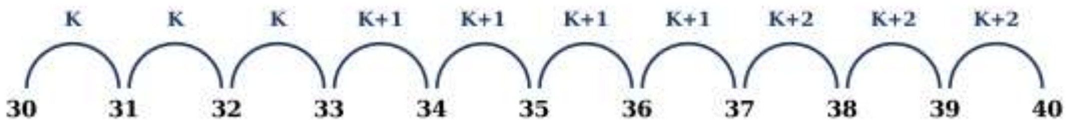

Figure 1: Finite-Difference Propagation Scheme over the Interval [30, 40]

Example of Propagation:

$$P(33) = S(30) + K + K + K \quad \text{or} \quad P(33) = S(35) - (K+1) - (K+1)$$

Note on Anchor Numbers of Form (10X + 5):
For anchor numbers of the form (10X + 5), the square is computed directly using:

$$(10X + 5)^2 = X(X + 1) \times 100 + 25$$

For example, the square of 35 is 3 × 4 | 25 = 1225, and the square of 45 is 4 × 5 | 25 = 2025.

NUMERICAL EXAMPLE
To demonstrate the application of the proposed method in practice, consider calculating the square of N = 43 using anchor A = 40:

Worked Example: Calculating 43^2 using Anchor Number A = 40	
Step 1: Input Deconstruction	N = 43 = 10X + Y • Tens component (X): 4 • Unit digit (Y): 3
Step 2: Unit Digit (Q)	Q ≡ Y^2 (mod 10) • Q = 3^2 = 9

Step 3: Anchor Selection & Base Prefix (S(A))	• Select Anchor Number: A = 40 • Square of Anchor: $A^2 = 40^2 = 1600$ **• Compute Base Prefix: S (40) = $\lfloor 1600 / 10 \rfloor$ = 160**
Step 4: Increment Constant (K)	$K = 2X = 2 \times 4 = 8$
Step 5: Stepwise Finite-Difference Propagation (P)	• Forward propagation sequence from 40 to 43 (+3 steps of K = 8): 40 —– (+8) —–> 41 —– (+8) —–> 42 —– (+8) —–> 43 • Calculate Target Prefix: P (43) = S (40) + 3K = 160 + 3(8) = 160 + 24 = 184
Step 6: Final Concatenation	$N^2 = 10P + Q = 10(184) + 9 = 1840 + 9 = 1849$ **• Result: $43^2 = 1849$**

4. ALGEBRAIC CONFIRMATION

Although this method was discovered empirically, its correctness can be confirmed algebraically. Writing

$$N = 10X + Y \quad (1)$$

we have:

$$N^2 = (10X + Y)^2 = 100X^2 + 20XY + Y^2 \quad (2)$$

As we know, for any integers a and b, [a + b] = a + [b] when a is an integer. Dividing Equation (2) by 10 and applying the greatest integer function gives:

$$P(N) = [N^2/10] = 10X^2 + 2XY + [Y^2/10] \quad (3)$$

Since $10X^2 + 2XY$ is an integer, it can be extracted from the floor function. Within a fixed interval $10X \leq N < 10(X+1)$, the value of X remains constant and the variation of P(N) as N increases is governed by changes in Y. In particular, the first difference is:

$$\Delta P(N) = P(N+1) - P(N) = 2X + [(Y+1)^2/10] - [Y^2/10] \quad (4)$$

The constant contribution 2X explains the observed base increment:

$$K = 2X \quad (5)$$

while occasional increases in the floor term $[(Y+1)^2/10] - [Y^2/10]$ account for the larger increments (K+1 and K+2) seen in the tabulated data. This confirms the empirically observed finite-difference structure without serving as its source.

Note on Multi-Digit Extension. Although presented using two-digit numbers for clarity, the method extends directly to multi-digit numbers where X represents all leading digits excluding the unit digit Y (for example, for N = 123, X = 12 and Y = 3, so K = 2X = 24).

CONCLUSION

This paper presents an additional method that attempts to reorganize squaring into smaller multiplications and sequential steps. Although all methods used for finding the squares of numbers are mathematically equivalent, they differ substantially in cognitive organization and pedagogical character. The proposed approach may therefore provide a useful supplementary learning pathway for students who find conventional symbolic or multiplication-based approaches cognitively demanding.

Declaration by Authors

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